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We investigate a two-stage serial supply chain with stationary stochastic demand and fixed transportation times. Inventory holding costs are charged at each stage, and each stage may incur a consumer backorder penalty cost, e.g. the upper stage (the supplier) may dislike backorders at the lower stage (the retailer). We consider two games. In both, the stages independently choose base stock policies to minimize their costs. The games differ in how the firms track their inventory levels (in one, the firms are committed to tracking echelon inventory; in the other they track local inventory). We compare the policies chosen under this competitive regime to those selected to minimize total supply chain costs, i.e., the optimal solution. We show that the games (nearly always) have a unique Nash equilibrium, and it differs from the optimal solution. Hence, competition reduces efficiency. Furthermore, the two games' equilibria are different, so the tracking method influences strategic behavior. We show that the system optimal solution can be achieved as a Nash equilibrium using simple linear transfer payments. The value of cooperation is context specific: In some settings competition increases total cost by only a fraction of a percent, whereas in other settings the cost increase is enormous. We also discuss Stackelberg equilibria.
(*Supply Chain*; *Game Theory*; *Multiechelon Inventory*; *Incentive Contracts*)

1. Introduction

How should a supply chain manage inventory? If the members care only about overall system performance, they should choose policies to minimize total costs, i.e., the optimal solution. While this approach is appealing, it harbors an important weakness. Each member may incur only a portion of the supply chain's costs, so the optimal solution may not minimize each member's own costs. For example, a supplier may care more than a retailer about consumer backorders for the supplier's product, or the retailer's cost to hold inventory may be higher than the supplier's. While the firms may agree in principal to cooperate, each may face a temptation to deviate from any agreement, to reduce its own costs. Supposing each firm can anticipate these temptations, how will the firms behave? Furthermore, to what extent will these temptations lead to supply chain inefficiency?

This paper studies the difference between global/cooperative and independent/competitive optimization in a serial supply chain with one supplier and one retailer. (We assume there are two independent firms, but the model also applies to independent agents within the same firm.) Consumer demand is stochastic, but independent and stationary across periods. There are inventory holding costs and consumer backorder penalty costs, but no ordering costs. There is a constant transportation time between stages, and the supplier's source has infinite capacity. Inventory is tracked using either *echelon inventory* or *local inventory*. (A firm's local inventory is its on-hand inventory, and its echelon inventory is its local inventory plus all

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inventory held lower in the supply chain.) In the optimal solution, the firms choose base stock policies, described in §3. These policies can be implemented by tracking either echelon inventory or local inventory.

To model independent decision making we consider two games, the Echelon Inventory (EI) game and the Local Inventory (LI) game. In both games the firms simultaneously choose their base stock levels. This is their only strategic decision, and it cannot be modified once it is announced. The supplier pays holding costs for inventory in its possession or in-transit to the retailer, and the retailer pays holding costs on units it possesses. Both firms are concerned about consumer backorders; the supplier pays a consumer backorder penalty as does the retailer. This is an important assumption, because it allows us to study how the firms' relative preferences influence their strategic behavior and, in turn, the performance of the system. (Section 3 discusses this modeling issue.)

The EI and LI games differ in only one way: In the EI game both firms are committed to tracking echelon inventory, whereas in the LI game both firms track local inventory.

The firms in each game play a Nash equilibrium. (A pair of strategies is a Nash equilibrium, if each firm minimizes its own cost assuming the other player chooses its equilibrium strategy.) Thus, each firm makes an optimal decision given the behavior of the other firm, and therefore neither firm has an incentive to deviate unilaterally from the equilibrium.

We find that in each game there is (usually) a unique Nash equilibrium. We compare the games' equilibria to each other and to the optimal solution. The optimal solution is typically *not* a Nash equilibrium, so competitive decision making degrades supply chain efficiency. We evaluate the magnitude of this effect with an extensive numerical study.

Implementation of the cooperative solution requires that the firms eliminate the incentives to deviate, i.e., they should modify their costs so that the optimal solution becomes a Nash equilibrium. This goal can be achieved by a contract that specifies linear transfer payments based on easily verifiable performance measures like inventory and backorders. We develop a set of linear contracts that meet this objective, and briefly discuss other techniques for aligning incentives.

In these games neither player dominates the other, and the firms simultaneously choose their strategies. We also study Stackelberg versions of the games, in which one dominant player chooses its strategy before the other.

The next section reviews the related literature, and §3 formulates the model. Section 4 describes the system optimal solution, and §5 analyzes the two games. Section 6 compares the games' equilibria with the optimal solution. Section 7 describes contracts that make the system optimal solution a Nash equilibrium. Section 8 discusses the numerical study. Section 9 analyzes the Stackelberg games, and §10 concludes.

2. Literature Review

The literature on supply chain inventory management mostly assumes policies are set by a central decision maker to optimize total supply chain performance. Three exceptions are Lee and Whang (1996), Chen (1997), and Porteus (1997).

In Lee and Whang (1996), the firms use echelon stock policies and all backorder penalties are charged to the lowest stage. The upper stage incurs holding costs only. Therefore, with competitive selection of policies, the upper stage carries no inventory, thereby minimizing its own cost. They develop a nonlinear transfer payment contract that induces each firm to choose the system optimal base stock policies. Our model differs from theirs on several dimensions. We assume the upper stage (the supplier) may care about consumer backorders, so it may carry inventory even when inventory policies are chosen competitively. Hence, the competitive decisions are nontrivial. We distinguish between echelon inventory and local inventory and investigate how these different methods for tracking inventory influence strategic behavior. Finally, we develop linear transfer payment contracts. They consider a setup cost at the upper echelon, while we do not. Porteus (1997) studies a model similar to Lee and Whang's model, but he proposes a different coordination scheme, called responsibility tokens.

Chen (1997) studies a game similar to the popular Beer Game (Sterman 1989), except that the demands in different periods are independent random variables with a common distribution that is known to all players. Unlike our players, his share the objective of minimizing total system costs; they have no competing interests. He outlines an accounting scheme that allows each player to optimize its own costs and yet choose the system optimal solution. This scheme is more complex than ours in some ways, though simpler in others, as explained in §7. He also studies the behavior of boundedly rational players, whereas we only assume rational players.

Several other papers address related issues, yet their models are significantly different. Lippman and McCardle (1997) study competition between two or more firms in a one-period setting, where a consumer may switch among firms to find available inventory. Parlar (1988) and Li (1992) also study the role of inventory in the competition among retailers. In a multiechelon model with multiple retailers, Muckstadt and Thomas (1980), Hausman and Erkip (1994), and Axsäter (1996) investigate a centralized control system that allows each firm to optimize its own costs and still choose an outcome desirable to the central planner. The behavior of a central planner has also been investigated in settings with moral hazard (e.g., Porteus and Whang 1991, Kouvelis and Lariviere 1996). Many papers investigate how a supplier can induce a retailer to behave in a manner that is more favorable to the supplier (e.g., Donohue 1996, Tsay 1996, Ha 1996, Lal and Staelin 1984, Moses and Seshadri 1996, Narayanan and Raman 1996, Pasternack 1985). Chen et al. (1997) study competitive selection of inventory policies in a multiechelon model with deterministic demand.

3. Model Description

Consider a one-product inventory system with one supplier and one retailer. The supplier is Stage 2 and the retailer is Stage 1. Time is divided into an infinite number of discrete periods. Consumer demand at the retailer is stochastic, independent across periods and stationary. The following is the sequence of events during a period: (1) shipments arrive at each stage; (2) orders are submitted and shipments are released; (3) consumer demand occurs; (4) holding and backorder penalty costs are charged.

There is a lead time for shipments from the source to the supplier, L_2, and from the supplier to the retailer, L_1. Each firm may order any nonnegative amount in each period. There is no fixed cost for placing or processing an order. Each firm pays a constant price per unit ordered, so there are no quantity discounts.

The supplier is charged holding cost h_2 per period for each unit in its stock or enroute to the retailer. The retailer's holding cost is $h_2 + h_1$ per period for each unit in its stock. Assume $h_2 > 0$ and $h_1 \geq 0$.

Unmet demands are backlogged, and all backorders are ultimately filled. Both the retailer and the supplier may incur costs when demand is backordered. The retailer is charged αp for each backorder, and the supplier $(1 - \alpha)p$, $0 \leq \alpha \leq 1$. The parameter p is the total system backorder cost, and α specifies how this cost is divided among the firms. The parameter α is exogenous.

These backorder costs have several possible interpretations, all standard. They may represent the costs of financing receivables, if customers pay only upon the fulfillment of demands. (This requires a discounted-cost model to represent exactly, but the approximation here is standard in the average-cost context, analogous to the treatment of inventory financing costs.) Alternatively, they may be proxies for losses in customer good-will, which in turn lead to long-run declines in demand. Such costs need not affect the firms equally, which is why we allow the flexibility to choose $\alpha \in [0, 1]$. Finally, they provide a crude approximation to lost sales. (It would be better, of course, to model lost sales directly, but that introduces considerable analytical difficulties. Even the optimal policy is unknown.)

In period t before demand define the following for stage i: *in-transit inventory*, IT_{it}; *echelon inventory level*, IL_{it}, is all inventory at stage i or lower in the system minus consumer backorders; *local inventory level*, $\overline{IL}_{it}$, is inventory at stage i minus backorders at stage i (the supplier's backorders are unfilled retailer orders); *echelon inventory position*, IP_{it}, $IP_{it} = IL_{it} + IT_{it}$; and *local inventory position*, $\overline{IP}_{it}$, $\overline{IP}_{it} = \overline{IL}_{it} + IT_{it}$.

Each firm uses a base stock policy. Using an echelon base stock level, each period the firm orders a sufficient amount to raise its echelon inventory position plus outstanding orders to that level. A firm's local base stock level is similar, except the local inventory position replaces the echelon inventory position. Define s_i as stage i's echelon base stock level and $\bar{s}_i$ as its local base stock level.

Let D^τ denote random total demand over τ periods, and μ^τ denote mean total demand over τ periods. Let ϕ^τ and Φ^τ be the density and distribution functions of demand over τ periods, respectively. We assume $\Phi^1(x)$ is continuous, increasing, and differentiable for $x \geq 0$, so the same is true of Φ^τ, $\tau > 0$. Furthermore, $\Phi^1(0) = 0$, so positive demand occurs in each period.

Math notation follows: $[x]^+ = \max\{0, x\}$; $[x]^- = \max\{0, -x\}$; $[a, b]$ is the closed interval from a to b; and $E[x]$ is the expected value of x. A prime denotes the derivative of a function of one variable.

4. System Optimal Solution

The system optimal solution minimizes the total average cost per period. Clark and Scarf (1960), Federgruen and Zipkin (1984), and Chen and Zheng (1994) demonstrate that an echelon base stock policy is optimal in this setting. The optimal solution is found by allocating costs to the firms in a particular way. Then, each firm chooses a policy that minimizes its cost function. This section briefly outlines this method.

Let $\hat{G}_1^o(IL_{1t} - D^1)$ equal the retailer's charge in period t, where

$$\hat{G}_1^o(x) = h_1[x]^+ + (h_2 + p)[x]^-.$$

Also in period t, define $G_1^o(IP_{1t})$ as the retailer's expected charge in period $t + L_1$, where

$$G_1^o(y) = E[\hat{G}_1^o(y - D^{L_1+1})].$$

Define s_1^o as the value of y that minimizes $G_1^o(y)$:

$$\Phi^{L_1+1}(s_1^o) = \frac{h_2 + p}{h_1 + h_2 + p}. \quad (1)$$

This is the retailer's optimal base stock level. Define the induced penalty function,

$$\underline{G}_1^o(y) = G_1^o(\min\{s_1^o, y\}) - G_1^o(s_1^o),$$

and define

$$\hat{G}_2^o(y) = h_2(y - \mu^1) + \underline{G}_1^o(y).$$

In period t charge the supplier $G_2^o(IP_{2t})$, where

$$G_2^o(y) = E[\hat{G}_2^o(y - D^{L_2})].$$

The supplier's optimal echelon base stock level, s_2^o, minimizes $G_2^o(\cdot)$.

5. Echelon and Local Inventory Games

In the Echelon Inventory (EI) game, the two stages are independent firms or players. In the game's only move, the players simultaneously choose their strategies, $s_i \in \sigma = [0, S]$, where s_i equals player i's echelon base stock level, σ is player i's strategy space, and S is a very large constant. (S is sufficiently large that it never constrains the players.) A joint strategy s is a pair (s_1, s_2). After their choices, the players implement their policies over an infinite horizon. In addition, all model parameters are common knowledge.

In the Local Inventory (LI) game the supplier and the retailer choose local base stock levels, $\bar{s}_2, \bar{s}_1 \in \sigma$. Again, strategies are chosen simultaneously, the players are committed to their strategies over an infinite horizon, and all parameters are common knowledge. The players know which game they are playing; the choice between the EI and LI games is not one of their decisions.

Define $H_i(s_1, s_2)$ as player i's expected per-period cost when players use echelon base stock levels (s_1, s_2). When $s_2 = \bar{s}_2 + \bar{s}_1$ and $s_1 = \bar{s}_1$, the local base stock pair $(\bar{s}_1, \bar{s}_2)$ is equivalent to (s_1, s_2) in the sense that $H_i(s_1, s_2) = H_i(\bar{s}_1, \bar{s}_2 + \bar{s}_1)$. Since any echelon base stock pair can be converted into an equivalent local pair, there is no need to define distinct cost functions with local arguments. We will frequently switch a pair of base stock levels from one tracking method to another to facilitate comparisons. Although there is little operational distinction between echelon and local base stock policies, we later show that they differ strategically. (However, the operational equivalence of echelon and local base stocks does depend on the assumption of stationary demand. In a nonstationary demand environment, it may not be possible to run the system optimally with local base stock policies.)

For the EI game the best reply mapping for firm i is a set-valued relationship associating each strategy s_j, $j \neq i$, with a subset of σ according to the following rules:

$$r_1(s_2) = \{s_1 \in \sigma \mid H_1(s_1, s_2) = \min_{x\in\sigma} H_1(x, s_2)\}$$

$$r_2(s_1) = \{s_2 \in \sigma \mid H_2(s_1, s_2) = \min_{x\in\sigma} H_2(s_1, x)\}.$$

Likewise, for the LI game, the best reply mappings are

$$\bar{r}_1(\bar{s}_2) = \{\bar{s}_1 \in \sigma \mid H_1(\bar{s}_1, \bar{s}_2 + \bar{s}_1) = \min_{x\in\sigma} H_1(x, \bar{s}_2 + x)\}$$

$$\bar{r}_2(\bar{s}_1) = \{\bar{s}_2 \in \sigma \mid H_2(\bar{s}_1, \bar{s}_2 + \bar{s}_1) = \min_{x\in\sigma} H_2(\bar{s}_1, x + \bar{s}_1)\}.$$

A pure strategy Nash equilibrium is a pair of echelon base stock levels, (s_1^e, s_2^e), in the EI game, or local base stock levels, $(\bar{s}_1^l, \bar{s}_2^l)$, in the LI game, such that each player chooses a best reply to the other player's equilibrium base stock level:

$$s_2^e \in r_2(s_1^e) \quad s_1^e \in r_1(s_2^e)$$

$$\bar{s}_2^l \in \bar{r}_2(\bar{s}_1^l) \quad \bar{s}_1^l \in \bar{r}_1(\bar{s}_2^l).$$

(We do not consider mixed strategies. We generally find a unique pure strategy equilibrium.)

5.1. Actual Cost Functions

In each period, the retailer is charged $h_1 + h_2$ per unit held in inventory and αp per unit backordered. Define $\hat{G}_1(IL_{1t} - D^1)$ as the sum of these costs in period t,

$$\hat{G}_1(y) = (h_1 + h_2)[y]^+ + \alpha p[y]^-.$$

Define $G_1(IP_{1t})$ as the retailer's expected cost in period $t + L_1$,

$$G_1(y) = E[\hat{G}_1(y - D^{L_1+1})]$$
$$= (h_1 + h_2)(y - \mu^{L_1+1}) + (h_1 + h_2 + \alpha p)$$
$$\times \int_y^\infty (x - y)\phi^{L_1+1}(x)dx.$$

Define s_1^a as the value that minimizes this function, that is, the base stock level that minimizes the retailer's costs, assuming retailer orders are shipped immediately,

$$s_1^a = \arg\min_{y\in\sigma} G_1(y).$$

Differentiation verifies that G_1 is strictly convex, so s_1^a is determined by $G_1'(s_1^a) = 0$,

$$\Phi^{L_1+1}(s_1^a) = \frac{\alpha p}{h_1 + h_2 + \alpha p}.$$

The retailer's true expected cost depends on both its own base stock as well as the supplier's base stock. We use a standard derivation. After the firms place their orders in period $t - L_2$, the supplier's echelon inventory position equals s_2. After inventory arrives in period t, but before period t demand, the supplier's echelon inventory level equals $s_2 - D^{L_2}$. Hence, $E[s_2 - D^{L_2}]$ is the supply chain's expected inventory level (average supply chain inventory minus average backorders). When $s_2 - D^{L_2} \geq s_1$, the supplier can completely fill the retailer's period t order, so $IP_{1t} = s_1$. When $s_2 - D^{L_2} < s_1$, the supplier cannot fill all of the retailer's order, and $IP_{1t} = s_2 - D^{L_2}$. Hence,

$$H_1(s_1, s_2) = E[G_1(\min\{s_2 - D^{L_2}, s_1\})]$$
$$= \Phi^{L_2}(s_2 - s_1)G_1(s_1)$$
$$+ \int_{s_2-s_1}^\infty \phi^{L_2}(x)G_1(s_2 - x)dx.$$

Define $\hat{G}_2(IL_{1t} - D^1)$ as the supplier's actual period t backorder cost,

$$\hat{G}_2(y) = (1 - \alpha)p[y]^-,$$

and $G_2(IP_{1t})$ as the supplier's expected period $t + L_1$ backorder cost,

$$G_2(y) = E[\hat{G}_2(y - D^{L_1+1})].$$

Define

$$\hat{H}_2(s_1, x) = h_2\mu^{L_1} + h_2[x]^+ + G_2(s_1 + \min\{x, 0\}),$$

so

$$H_2(s_1, s_2) = E[\hat{H}_2(s_1, s_2 - s_1 - D^{L_2})]$$

$$= h_2\mu^{L_1} + h_2 \int_0^{s_2-s_1} (s_2 - s_1 - x)\phi^{L_2}(x)dx$$

$$+ \Phi^{L_2}(s_2 - s_1)G_2(s_1)$$

$$+ \int_{s_2-s_1}^{\infty} \phi^{L_2}(x)G_2(s_2 - x)dx.$$

The first term above is the expected holding cost for the units in-transit to the retailer (from Little's Law), the second term is the expected cost for inventory held at the supplier and the final two terms are the expected backorder cost charged to the supplier.

We mentioned above the operational equivalence of local and echelon base stock policies when $s_1 = \bar{s}_1$ and $s_2 = \bar{s}_1 + \bar{s}_2$. However, the *change* in player *i*'s cost due to a shift in player *j*'s strategy depends on the inventory tracking method. For example, holding $\bar{s}_2$ constant, the supplier's expected on-hand inventory is independent of $\bar{s}_1$, but when s_2 stays constant, the supplier's inventory declines as s_1 increases. Furthermore, the total system inventory depends on s_2 only. So holding s_2 fixed, the retailer's s_1 only influences the allocation of inventory between the supplier and the retailer. However, holding $\bar{s}_2$ fixed, the retailer can increase total system inventory by raising $\bar{s}_1$.

5.2. Echelon Inventory Game Equilibria with Shared Backorder Costs

In this section, we assume that each firm incurs some backorder cost, i.e., $0 < \alpha < 1$. (We subsequently consider the extreme cases $\alpha = 0$ and $\alpha = 1$.) We begin with some preliminary results on the players' cost functions and best reply mappings.

LEMMA 1. *Assuming $\alpha < 1$, $H_2(s_1, s_2)$ is strictly convex in s_2, $s_2 \geq 0$, and $H_1(s_1, s_2)$ is quasiconvex in s_1.*

PROOF. Fix D^{L_2} and s_1. Consider the following function of s_2:

$$h_2(s_2 - s_1 - D^{L_2})^+ + G_2(\min\{s_2 - D^{L_2}, s_1\}).$$

Both terms are convex, while the second term is strictly convex in the interval $s_2 \in [D^{L_2}, D^{L_2} + s_1]$. Now take the expectation over D^{L_2}. The first term, $h_2E[(s_2 - s_1 - D^{L_2})^+]$, is convex, and strictly convex for $s_2 \geq s_1$. The second term, $E[G_2(\min\{s_2 - D^{L_2}, s_1\})]$, is convex, and strictly convex when $s_1 \geq 0$ and $s_2 \geq 0$. Hence, $H_2(s_1, s_2)$ is strictly convex in $s_2 \geq 0$.

Consider H_1. When $s_1 \geq s_2$, H_1 is constant with respect to s_1. Assume $s_1 < s_2$ and differentiate H_1,

$$\frac{\partial H_1}{\partial s_1} = \Phi^{L_2}(s_2 - s_1)G_1'(s_1).$$

When $s_2 \leq s_1^a$, H_1 is decreasing for $s_1 < s_2$ and constant for $s_1 \geq s_2$. When $s_2 > s_1^a$, H_1 is decreasing for $s_1 < s_1^a$, increasing for $s_1^a \leq s_1 \leq s_2$, and constant for $s_1 > s_2$. Hence, H_1 is quasiconvex in s_1. □

The following lemma characterizes the supplier's best reply mapping.

LEMMA 2. *Assuming $\alpha < 1$, $r_2(s_1)$ is a function, $r_2(s_1) > s_1$, and $0 < r_2'(s_1) < 1$.*

PROOF. From Lemma 1, H_2 is strictly convex in s_2, so $r_2(s_1)$ is a function (i.e., H_2 has a unique minimum) and is determined by the first-order condition

$$\frac{\partial H_2}{\partial s_2} = h_2\Phi^{L_2}(s_2 - s_1) + \int_{s_2-s_1}^{\infty} \phi^{L_2}(x)G_2'(s_2 - x)dx = 0.$$

This condition cannot hold at $s_2 \leq s_1$ because then $\Phi^{L_2}(s_2 - s_1) = 0$ and $G_2'(y) < 0$. Therefore, $s_2 = r_2(s_1) > s_1$. Given $s_2 > s_1$, from the implicit function theorem,

$$r_2'(s_1) = -\left(\frac{\partial^2 H_2}{\partial s_2 \partial s_1}\Big/\frac{\partial^2 H_2}{\partial s_2^2}\right)$$

$$= \frac{-\dfrac{\partial^2 H_2}{\partial s_2 \partial s_1}}{-\dfrac{\partial^2 H_2}{\partial s_2 \partial s_1} + \int_{s_2-s_1}^{\infty} \phi^{L_2}(x)G_2''(s_2 - x)dx}, \quad (2)$$

where

$$\frac{\partial^2 H_2}{\partial s_2^2} = (h_2 - G_2'(s_1))\phi^{L_2}(s_2 - s_1)$$

$$+ \int_{s_2-s_1}^{\infty} \phi^{L_2}(x)G_2''(s_2 - x)dx,$$

$$\frac{\partial^2 H_2}{\partial s_2 \partial s_1} = -\phi^{L_2}(s_2 - s_1)(h_2 - G_2'(s_1)).$$

The cross partial of H_2 is negative because $G_2' < 0 < h_2$ and $\phi^{L_2}(s_2 - s_1) > 0$ for $s_2 > s_1$. Since $G_2'' > 0$, $0 < r_2'(s_1) < 1$. □

Although in the EI game the supplier does not always fill the retailer's orders immediately, the supplier's echelon base stock level has little influence over the retailer's strategy.

LEMMA 3. *For the EI game, the retailer's best reply mapping is*

$$r_1(s_2) = \begin{cases} s_1^a & s_2 > s_1^a \\ [s_2, S] & s_2 \le s_1^a \end{cases}.$$

PROOF. Recall that $G_1(y)$ is strictly convex and minimized by $y = s_1^a$. Let $x = D^{L_2}$. When $s_2 - x \le s_1^a$, $s_1 \ge s_2$ minimizes $G_1(\min\{s_2 - x, s_1\})$. When $s_2 - x > s_1^a$, only $s_1 = s_1^a$ minimizes $G_1(\min\{s_2 - x, s_1\})$. When $s_2 \le s_1^a$, $s_2 - D^{L_2} \le s_1^a$, so $r_1(s_2) = [s_2, S]$. When $s_2 > s_1^a$, only $s_1 = s_1^a$ minimizes $G_1(\min\{s_2 - x, s_1\})$ for all x, so $r_1(s_2) = s_1^a$. □

The retailer's best reply is not necessarily unique, but there is only one Nash equilibrium.

THEOREM 4. *Assuming* $0 < \alpha < 1$, *in the EI game* ($s_1^e = s_1^a$, $s_2^e = r_2(s_1^a)$) *is the unique Nash equilibrium.*

PROOF. From Theorem 1.2 in Fudenberg and Tirole (1991), a pure strategy Nash equilibrium exists if (1) each player's strategy space is a nonempty, compact convex subset of a Euclidean space, and (2) player i's cost function is continuous in s and quasiconvex in s_i. By the assumptions and Lemma 1, these conditions are met, so there is at least one equilibrium. From Lemma 2 in any equilibrium, (s_1^e, s_2^e), $s_2^e = r_2(s_1^e) > s_1^e$. If $s_2^e \le s_1^a$, Lemma 3 implies $s_1^e \ge s_2^e$, a contradiction. Hence $s_2^e > s_1^a$, but from Lemma 3, this implies $s_1^e = s_1^a$. Since r_2 is a function, there is only one $s_2^e = r_2(s_1^a)$. Therefore, the equilibrium is unique. □

Figures 1 and 2 plot the firms' reaction functions and the resulting equilibrium for two examples.

5.3. Local Inventory Game Equilibria with Shared Backorder Costs

The analysis of the LI game also begins by characterizing the cost functions and the best reply mappings.

Figure 1 **Reaction Functions,** $\alpha = 0.30$, $p = 5$, $h_1 = h_2 = 0.5$, $L_1 = L_2 = 1$

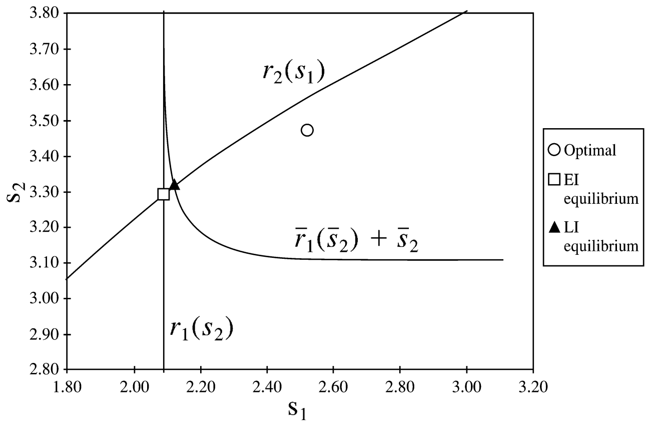
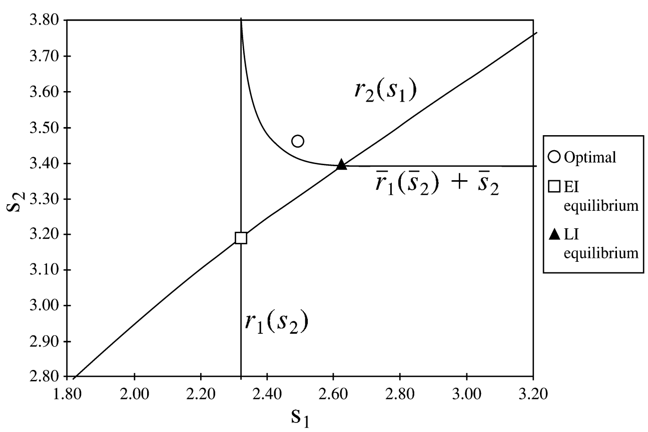


Figure 2 **Reaction Functions,** $\alpha = 0.90$, $p = 5$, $h_1 = h_2 = 0.5$, $L_1 = L_2 = 1$



LEMMA 5. $H_2(\bar{s}_1, \bar{s}_1 + \bar{s}_2)$ *is strictly convex in* $\bar{s}_2$ *and* $H_1(\bar{s}_1, \bar{s}_1 + \bar{s}_2)$ *is strictly convex in* $\bar{s}_1$.

PROOF. Set $s_2 = \bar{s}_1 + \bar{s}_2$ and $s_1 = \bar{s}_1$. Differentiation of $H_2(\bar{s}_1, \bar{s}_1 + \bar{s}_2)$ reveals that

$$\frac{\partial H_2(\bar{s}_1, \bar{s}_1 + \bar{s}_2)}{\partial \bar{s}_2} = \frac{\partial H_2(s_1, s_2)}{\partial s_2};$$

$$\frac{\partial^2 H_2(\bar{s}_1, \bar{s}_1 + \bar{s}_2)}{\partial \bar{s}_2^2} = \frac{\partial^2 H_2(s_1, s_2)}{\partial s_2^2}. \tag{3}$$

From Lemma 1, $H_2(s_1, s_2)$ is strictly convex in s_2, so $H_2(\bar{s}_1, \bar{s}_1 + \bar{s}_2)$ is strictly convex in $\bar{s}_2$. Differentiate $H_1(\bar{s}_1, \bar{s}_1 + \bar{s}_2)$,

$$\frac{\partial H_1(\bar{s}_1, \bar{s}_1 + \bar{s}_2)}{\partial \bar{s}_1} = \Phi^{L_2}(\bar{s}_2)G_1'(\bar{s}_1) + \int_{\bar{s}_2}^{\infty} \phi^{L_2}(x)G_1'(\bar{s}_1 + \bar{s}_2 - x)dx;$$

$$\frac{\partial^2 H_1(\bar{s}_1, \bar{s}_1 + \bar{s}_2)}{\partial \bar{s}_1^2} = \Phi^{L_2}(\bar{s}_2)G_1''(\bar{s}_1) + \int_{\bar{s}_2}^{\infty} \phi^{L_2}(x)G_1''(\bar{s}_1 + \bar{s}_2 - x)dx. \quad (4)$$

Since $G_1(\cdot)$ is strictly convex, $\partial^2 H_1(\bar{s}_1, \bar{s}_1 + \bar{s}_2)/\partial \bar{s}_1^2 > 0$, which means that H_1 is strictly convex in $\bar{s}_1$. □

The next two lemmas characterize the best reply mappings.

Lemma 6. *Assuming* $\alpha < 1$, $\bar{r}_2(s_1) + s_1 = r_2(s_1)$; $\bar{r}_2(s_1) > 0$; *and* $-1 < \bar{r}_2'(s_1) < 0$.

Proof. For the supplier $\bar{r}_2(s_1) + s_1 = r_2(s_1)$, because $H_2(\bar{s}_1, \bar{s}_1 + \bar{s}_2) = H_2(s_1, s_2)$ whenever $s_2 = \bar{s}_1 + \bar{s}_2$ and $s_1 = \bar{s}_1$. From Lemma 2, $r_2(s_1) > s_1$, which implies that $\bar{r}_2(s_1) > 0$. From the same lemma, $0 < r_2'(s_1) < 1$. Also $\bar{r}_2'(s_1) = r_2'(s_1) - 1$, so $-1 < \bar{r}_2'(s_1) < 0$. □

Lemma 7. *Assuming* $\alpha > 0$, $\bar{r}_1(\bar{s}_2) > s_1^a$. *When* $\bar{s}_2 > 0$, $-1 < \bar{r}_1'(\bar{s}_2) < 0$; *and when* $\bar{s}_2 = 0$, $\bar{r}_1'(\bar{s}_2) = -1$.

Proof. The retailer's best reply is determined by the first order condition $\partial H_1(\bar{s}_1, \bar{s}_1 + \bar{s}_2)/\partial \bar{s}_1 = 0$, see (4). When $\bar{s}_1 \le s_1^a$, $G_1'(\bar{s}_1) < 0$, and therefore $\partial H_1(\bar{s}_1, \bar{s}_1 + \bar{s}_2)/\partial \bar{s}_1 < 0$. Hence $\bar{r}_1(\bar{s}_2) > s_1^a$. From the implicit function theorem

$$\bar{r}_1'(\bar{s}_2) = -\frac{\partial^2 H_1(\bar{s}_1, \bar{s}_1 + \bar{s}_2)}{\partial \bar{s}_1 \partial \bar{s}_2} \Big/ \frac{\partial^2 H_1(\bar{s}_1, \bar{s}_1 + \bar{s}_2)}{\partial \bar{s}_1^2} = -\frac{\int_{\bar{s}_2}^{\infty} \phi^{L_2}(x)G_1''(\bar{s}_1 + \bar{s}_2 - x)dx}{\Phi^{L_2}(\bar{s}_2)G_1''(\bar{s}_1) + \int_{\bar{s}_2}^{\infty} \phi^{L_2}(x)G_1''(\bar{s}_1 + \bar{s}_2 - x)dx}.$$

Assume $\bar{s}_2 > 0$. Since $\Phi^{L_2}(\bar{s}_2)G_1''(\bar{s}_1) > 0$, the numerator above is positive and the numerator equals the second term in the denominator, $-1 < \bar{r}_1'(\bar{s}_2) < 0$. When $\bar{s}_2 = 0$, $\Phi^{L_2}(\bar{s}_2)G_1''(\bar{s}_1) = 0$ and therefore $\bar{r}_1'(\bar{s}_2) = -1$. □

When $0 < \alpha < 1$, there is a unique Nash equilibrium in the LI game.

Theorem 8. *Assuming* $0 < \alpha < 1$, $(\bar{s}_1^l, \bar{s}_2^l)$ *is the unique Nash equilibrium.*

Proof. Lemma 5 confirms the required conditions for the existence of an equilibrium (in the proof of Theorem 4). First, from Lemma 6, $\bar{r}_2(s_1) = r_2(s_1) - s_1 > 0$, so $\bar{s}_2^l > 0$. Now, suppose there are two equilibria, $(\bar{s}_1^l, \bar{s}_2^l)$ and $(\bar{s}_1^*, \bar{s}_2^*)$. Without loss of generality, assume $\bar{s}_2^l < \bar{s}_2^*$. From Lemma 7, this implies that $\bar{s}_1^* < \bar{s}_1^l$. From the same lemma, $\bar{r}_1'(\bar{s}_2) > -1$, so $s_2^* = \bar{s}_1^* + \bar{s}_2^* > \bar{s}_1^l + \bar{s}_2^l = s_2^l$. But from Lemma 2, r_2 is increasing, so $s_2^* > s_2^l$ implies that $\bar{s}_1^* > \bar{s}_1^l$, a contradiction. Hence, there is a unique equilibrium. □

Figures 1 and 2 also display the reaction functions in the LI game as well as the Nash equilibrium.

5.4. Equilibria Under Extreme Backorder Cost Allocations

Suppose the retailer is charged all of the backorder costs, i.e., $\alpha = 1$. In this situation, the Nash equilibrium in the EI game is no longer unique.

Theorem 9. *For* $\alpha = 1$, *in the EI game the Nash equilibria are* $(s_1^e \in [s_2^e, S], s_2^e \in [0, s_1^a])$.

Proof. The existence proof in Theorem 4 applies even when $\alpha = 1$, so a pure strategy equilibrium exists. When $\alpha = 1$, the supplier incurs no backorder costs, only holding costs. Hence, the supplier picks $s_2 \le s_1$, i.e., $r_2(s_1) = [0, s_1]$. Suppose (s_1^*, s_2^*) is an equilibrium, where $s_2^* > s_1^a$. From Lemma 3, $r_1(s_2^*) = s_1^a$, but an equilibrium only occurs when $s_2^* \le s_1^*$, so $s_2^* > s_1^a$ cannot be an equilibrium. Suppose $s_2^* \le s_1^a$. From Lemma 3, $r_1(s_2^*) = [s_2^*, S]$, so for any $s_2^* \le s_1^a$, $(s_1^* \in [s_2^*, S], s_2^*)$ is an equilibrium. □

In the LI game there is a unique equilibrium even when the retailer incurs all of the backorder cost.

Theorem 10. *Assuming* $\alpha = 1$, *in the LI game* $(\bar{s}_1^l = \bar{r}_1(0), \bar{s}_2^l = 0)$ *is the unique Nash equilibrium.*

Proof. When $\alpha = 1$ the supplier chooses $\bar{s}_2 = 0$. Since $\bar{r}_1(\bar{s}_2)$ is a function, $\bar{r}_1(0)$ is unique. □

When the supplier incurs all backorder costs, there is a unique equilibrium in both games and they are identical.

THEOREM 11. *Assuming* $\alpha = 0$, $(s_1^e = 0, s_2^e = r_2(0))$ *is the unique Nash equilibrium in the EI game, and* $(\bar{s}_1^l = s_1^e, \bar{s}_2^l = s_2^e - \bar{s}_1^l)$ *is the unique Nash equilibrium of the LI game.*

PROOF. Since the retailer incurs no backorder cost $s_1^e = \bar{s}_1^l = 0$. The supplier's best reply mapping is a function in either game, so $s_2^e = r_2(0)$. Furthermore, $\bar{s}_2^l = s_2^e - s_1^e$. □

6. Comparing Equilibria

This section compares the equilibria in the LI and EI games to each other as well as to the optimal solution. To facilitate these comparisons, convert the LI game equilibrium, $(\bar{s}_1^l, \bar{s}_2^l)$, into the equivalent pair of echelon base stock levels, (s_1^l, s_2^l), where $s_1^l = \bar{s}_1^l$ and $s_2^l = \bar{s}_1^l + \bar{s}_2^l$.

6.1. Competitive Equilibria

The firms choose higher base stock levels in the LI game than in the EI game.

THEOREM 12. *Assuming* $0 < \alpha < 1$, *the base stock levels for both firms are higher in the LI game equilibrium than in the EI game equilibrium, i.e.,* $s_2^l > s_2^e$ *and* $s_1^l > s_1^e$.

PROOF. The equilibrium in the EI game is $(s_1^e = s_1^a, s_2^e = r_2(s_1^e))$. From Lemma 7, $\bar{r}_1(\bar{s}_2) > s_1^a$, which implies that $s_1^l > s_1^e = s_1^a$. From Lemma 2, $r_2(s_1)$ is increasing in s_1, so $s_2^l = r_2(s_1^l) > r_2(s_1^a) = s_2^e$. □

The retailer's cost in the LI game equilibrium can be more or less than in the EI game equilibrium. (The numerical study confirms this.) However, the supplier has a definite preference for the LI game.

THEOREM 13. *Assuming* $0 < \alpha < 1$, *the supplier's cost in the LI game equilibrium is lower than its cost in the EI game equilibrium.*

PROOF. In the EI game the supplier chooses $r_2(s_1)$. In the LI game the supplier chooses $r_2(s_1) - s_1$ as its local base stock level and the equivalent echelon base stock level is $r_2(s_1)$. Differentiate the supplier's cost function with respect to the retailer's base stock level, assuming the supplier chooses $s_2 = r_2(s_1)$:

$$\frac{dH_2}{ds_1} = \frac{\partial H_2}{\partial s_1} + \frac{\partial H_2}{\partial s_2}\frac{\partial r_2(s_1)}{\partial s_1} = \frac{\partial H_2}{\partial s_1}$$
$$= -\Phi^{L_2}(s_2 - s_1)(h_2 - G_2'(s_1)),$$

since $\partial H_2(s_1, r_2(s_1))/\partial s_2 = 0$. From Lemma 2, $r_2(s_1) > s_1$, so $\Phi^{L_2}(s_2 - s_1) > 0$, and $G_2' < 0$, so $dH_2/ds_1 < 0$. Thus, the supplier's cost declines as s_1 increases. Since $s_1^l > s_1^e$, H_2 is lower at s_1^l. □

Why does the supplier prefer the LI game equilibrium? The supplier always prefers the retailer to increase its base stock, thereby increasing the retailer's inventory and decreasing the supplier's backorder costs. The retailer always chooses a lower base stock in the EI game than it does in the LI game, hence the supplier is always better off in the LI game.

6.2. Competitive Equilibria and the Optimal Solution

In the EI game the retailer's base stock level is lower than in the optimal solution.

THEOREM 14. *In an EI game equilibrium, the retailer's base stock level is lower than in the optimal solution.*

PROOF. Note that $\hat{G}_1^{o\prime}(x) < \hat{G}_1'(x)$, for all x. Hence, $G_1^{o\prime}(y) < G_1'(y)$, for all y. Since both $G_1^{o\prime}(y)$ and $G_1'(y)$ are increasing in y, $s_1^o > s_1^a = s_1^e$. □

In the LI game either $s_1^l > s_1^o$ or $s_1^l < s_1^o$ is possible. (In Figure 1 the retailer chooses $s_1^l < s_1^o$, but in Figure 2 $s_1^l > s_1^o$.) However, when backorder costs are charged to the supplier, the supplier's base stock level is lower than in the system optimal solution in both games.

THEOREM 15. *Assuming* $\alpha < 1$, *the supplier's base stock level in both the LI and the EI equilibria is lower than in the system optimal solution.*

PROOF. In any equilibrium $s_2^l > s_2^e$, so it is sufficient to show that $s_2^o > s_2^l$. For $x < -s_1$,

$$\frac{\partial \hat{H}_2}{\partial x} = G_2'(s_1 + x) = -(1 - \alpha)p$$

and

$$\hat{G}_2^{o\prime}(x) = h_2 + \underline{G}_1^{o\prime}(x) = h_2 + G_1^{o\prime}(x)$$
$$= -p.$$

For $-s_1 < x < 0$

$$\frac{\partial \hat{H}_2}{\partial x} = G_2'(s_1 + x) \in (-(1-\alpha)p, 0]$$

and again $\hat{G}_2^{o\prime}(x) = -p$. For $x > 0$, $\partial \hat{H}_2/\partial x = h_2$ and

$$\hat{G}_2^{o\prime}(x) = h_2 + G_1^{o\prime}(x) \le h_2,$$

with strict inequality for $x < s_1^o$. So in all cases $\hat{G}_2^{o\prime}(x) \le \partial \hat{H}_2/\partial x$, with strict inequality for $-s_1 < x < s_1^o$. Therefore, $G_2^{o\prime}(x) \le \partial H_2/\partial x$, with strict inequality for $s_2 > s_1$. So, $s_2^o > s_2^l$. □

Recall that the supplier's echelon base stock determines the supply chain's average inventory level. From Theorem 15 it follows that in either game's equilibrium the supply chain's average inventory level will be lower than in the optimal solution, suggesting that competition will also tend to lower the supply chain's average inventory. The numerical study confirms this observation.

When the supplier incurs no backorder costs, the supplier's base stock level is no greater than in the system optimal solution.

THEOREM 16. *Assuming* $\alpha = 1$, $s_2^e < s_2^o$, *and* $s_2^l \le s_2^o$.

PROOF. In the EI game $s_2^e = s_1^e \le s_1^a < s_1^o \le s_2^o$, hence $s_2^e < s_2^o$. When $\alpha = 1$, in the LI game $s_2^l = s_1^l$, so the proof of Theorem 15 demonstrates $s_2^l \le s_2^o$. □

THEOREM 17. *Assuming* $\alpha < 1$, *the system optimal solution is not a Nash equilibrium.*

PROOF. From Theorem 15, $s_2^e \le s_2^l < s_2^o$, so the optimal solution is not a Nash equilibrium in either game. □

When the supplier incurs no backorder costs, the system optimal solution can be a Nash equilibrium under a *very* special condition.

THEOREM 18. *Assuming* $\alpha = 1$, *the system optimal solution is a Nash equilibrium in the LI game only when*

$$\Phi^{L_2+L_1+1}(s_1^o) = \frac{p}{h_1 + h_2 + p}.$$

PROOF. When $\alpha = 1$, the LI game Nash equilibrium is $(s_1^l, s_2^l = s_1^l)$. Solving for s_1^l,

$$\Phi^{L_2+L_1+1}(s_1^l) = \frac{p}{h_1 + h_2 + p}.$$

It is possible that $s_1^l = s_1^o$ because $\Phi^{L_2+L_1+1}$ stochastically dominates Φ^{L_1+1} and $p/(h_1 + h_2 + p) < (h_2 + p)/(h_1 + h_2 + p)$. For the supplier $s_2^o = s_1^o$ when $G_2^{o\prime}(s_2^o = s_1^o) = 0$. This occurs precisely when

$$\Phi^{L_2+L_1+1}(s_1^o) = \frac{p}{h_1 + h_2 + p}.$$

In that case, $s_2^o = s_1^o = s_1^l = s_2^l$. □

7. Cooperative Inventory Policies

According to Theorem 17, the optimal solution is virtually never a Nash equilibrium. Hence, the firms can lower total costs by acting cooperatively.

There are several methods that enable the firms to minimize total costs and still remain confident that the other firm will not deviate from this agreement. For instance, the firms could contract to choose (s_1^o, s_2^o) as their base stock levels. But, since each firm has an incentive to deviate from this contract (because it is not a Nash equilibrium), the contract must also specify a penalty for deviations. Such stipulations are hard to enforce.

Alternatively, the firms could write a contract that specifies transfer payments which eliminate incentives to deviate from the optimal solution. There are several schemes to achieve this goal: a per period fee for the supplier's backorder; a per unit fee for each unit the supplier does not ship immediately; a per unit fee per consumer backorder; or a subsidy for each unit of inventory in the system. (Nonlinear payment schedules, as in Lee and Whang (1996), could also be considered, but these are necessarily more cumbersome. For instance, the induced penalty function $\underline{G}_1^o$ is nonlinear.) Transfer payments can also be imposed on the retailer, e.g. a subsidy on retailer inventories, or a subsidy on backorders.

Payments could also be based on the inventory and backorder levels that *would have* occurred had the supplier performed certain actions. Chen (1997) uses this approach. Define accounting inventory and accounting backorders as the inventory and backorder levels, assuming the supplier fills retailer orders immediately. Suppose the supplier pays all of the retailer's actual costs, and the supplier charges the retailer h_1 per unit of accounting inventory and $h_2 + p$ per accounting backorder. Then, the retailer will choose s_1^o. Since the supplier incurs all actual costs, and the retailer chooses s_1^o, the supplier chooses s_2^o. With this scheme, the retailer's decision is independent of the supplier's; there is no strategic interaction between the firms. However, this approach creates a challenging accounting problem.

We study linear transfer payments based on actual inventory and backorder levels. While our approach avoids the problem of tracking accounting inventory and backorders, Chen's method uses only cost parameters. Ours also requires a demand parameter. Thus, his technique may be easier to implement in some situations, ours in others.

7.1. Linear Contracts

Suppose the firms track local inventory and they adopt a transfer payment contract with constant parameters $(\iota_1, \beta_2, \beta_1)$. This contract specifies that the period t transfer payment from the supplier to the retailer is

$$\iota_1 I_{1t} + \beta_2 B_{2t} + \beta_1 B_{1t},$$

where I_{1t} is the retailer's on-hand inventory, and B_{it} is stage i's backorders, all measured at the end of the period. There are no *a priori* sign restrictions on these parameters, e.g. $\iota_1 > 0$ represents a holding cost subsidy to the retailer and $\iota_1 < 0$ represents a holding fee. (We later impose some restrictions on the parameters.) We also assume the optimal solution is common knowledge.

Define $T_1(IP_{1t})$ as the expected transfer payment in period $t + L_1$ due to retailer inventory and backorders, where

$$\begin{aligned} T_1(y) &= E[\iota_1[y - D^{L_1+1}]^+ + \beta_1[y - D^{L_1+1}]^-] \\ &= \iota_1(y - \mu^{L_1+1}) \\ &\quad + (\iota_1 + \beta_1)\int_y^\infty (x - y)\phi^{L_1+1}(x)dx. \end{aligned}$$

Define $T(\bar{s}_1, \bar{s}_2)$ as the expected per period transfer payment from the supplier to the retailer,

$$\begin{aligned} T(\bar{s}_1, \bar{s}_2) &= E[\beta_2[\bar{s}_2 - D^{L_2}]^- \\ &\quad + T_1(\bar{s}_1 + \min\{0, \bar{s}_2 - D^{L_2}\})] \\ &= \beta_2 \int_{\bar{s}_2}^\infty \phi^{L_2}(x)(x - \bar{s}_2)dx + \Phi^{L_2}(\bar{s}_2)T_1(\bar{s}_1) \\ &\quad + \int_{\bar{s}_2}^\infty \phi^{L_2}(x)T_1(\bar{s}_1 + \bar{s}_2 - x)dx. \end{aligned}$$

Note that $\bar{s}_1$ influences the retailer inventory and backorders, but not the supplier's backorders. Let $H_i^c(\bar{s}_1, \bar{s}_1 + \bar{s}_2)$ be player i's costs after accounting for the transfer payment,

$$H_1^c(\bar{s}_1, \bar{s}_1 + \bar{s}_2) = H_1(\bar{s}_1, \bar{s}_1 + \bar{s}_2) - T(\bar{s}_1, \bar{s}_2),$$

$$H_2^c(\bar{s}_1, \bar{s}_1 + \bar{s}_2) = H_2(\bar{s}_1, \bar{s}_1 + \bar{s}_2) + T(\bar{s}_1, \bar{s}_2).$$

We wish to determine the set of contracts, $(\iota_1, \beta_2, \beta_1)$, such that $(\bar{s}_1^c, \bar{s}_2^c)$ is a Nash equilibrium for the cost functions $H_i^c(\bar{s}_1, \bar{s}_1 + \bar{s}_2)$, where $\bar{s}_1^c = s_1^o$, and $\bar{s}_1^c + \bar{s}_2^c = s_2^o$. With these contracts the firms can choose $(\bar{s}_1^c, \bar{s}_2^c)$, thereby minimizing total costs, and also be assured that no player has an incentive to deviate.

To find the desired set of contracts, first assume that H_i^c is strictly convex in $\bar{s}_i$, given that player j chooses $\bar{s}_j^c$, $j \neq i$. Then determine the contracts in which $\bar{s}_i^c$ satisfies player i's first order condition, thereby minimizing player i's cost. Finally, determine the subset of these contracts that also satisfy the original strict convexity assumption.

The following are the first order conditions:

$$\begin{aligned} \frac{\partial H_1^c}{\partial \bar{s}_1} = 0 &= \Phi^{L_2}(\bar{s}_2)(G_1'(\bar{s}_1) - T_1'(\bar{s}_1)) \\ &\quad + \int_{\bar{s}_2}^\infty \phi^{L_2}(x)[G_1'(\bar{s}_1 + \bar{s}_2 - x) \\ &\qquad - T_1'(\bar{s}_1 + \bar{s}_2 - x)]dx; \end{aligned} \tag{5}$$

$$\begin{aligned} \frac{\partial H_2^c}{\partial \bar{s}_2} = 0 &= -\beta_2 + (h_2 + \beta_2)\Phi^{L_2}(\bar{s}_2) \\ &\quad + \int_{\bar{s}_2}^\infty \phi^{L_2}(x)[G_2'(\bar{s}_1 + \bar{s}_2 - x) \end{aligned}$$

$$+ T_1'(\bar{s}_1 + \bar{s}_2 - x)]dx. \quad (6)$$

Define $\gamma_2 = \Phi^{L_2}(s_2^o - s_1^o)$. (This is the supplier's in-stock probability, essentially its fill rate.) Furthermore, the supplier's first order condition in the optimal solution is

$$0 = -p + (p + h_2)\Phi^{L_2}(s_2^o - s_1^o) + (h_1 + h_2 + p) \times \int_{s_2^o - s_1^o}^{\infty} \phi^{L_2}(x)\Phi^{L_1+1}(s_2^o - x)dx,$$

or,

$$\int_{s_2^o - s_1^o}^{\infty} \phi^{L_2}(x)\Phi^{L_1+1}(s_2^o - x)dx = \frac{p - (p + h_2)\gamma_2}{h_1 + h_2 + p}. \quad (7)$$

Using (7), (5), and (6) yield the following two equations in three unknowns,

$$(1 - \alpha)p = \left(\frac{p}{h_1 + h_2}\right)\iota_1 - \beta_1, \quad (8)$$

$$h_2 = \left(\frac{h_2}{h_1 + h_2}\right)\iota_1 + \left(\frac{1 - \gamma_2}{\gamma_2}\right)\beta_2. \quad (9)$$

It remains to ensure that the costs functions are indeed strictly convex.

Theorem 19. *When the firms choose* $(\iota_1, \beta_2, \beta_1)$ *to satisfy* (8) *and* (9), *and the following additional restrictions apply*

(i) $h_1 + h_2 > \iota_1 \geq 0$

(ii) $\beta_2 > 0$

(iii) $\alpha p > \beta_1 \geq -(1 - \alpha)p$,

then the optimal policy $(\bar{s}_1^c, \bar{s}_2^c)$ *is a Nash equilibrium.*

Proof. When the following second order conditions are satisfied, H_i^c is strictly convex in $\bar{s}_i$, assuming $\bar{s}_j = \bar{s}_j^c$, $j \neq i$:

$$\frac{\partial^2 H_1^c}{\partial \bar{s}_1^2} = \Phi^{L_2}(\bar{s}_2)(G_1''(\bar{s}_1) - T_1''(\bar{s}_1)) + \int_{\bar{s}_2}^{\infty} \phi^{L_2}(x)(G_1''(\bar{s}_1 + \bar{s}_2 - x) - T_1''(\bar{s}_1 + \bar{s}_2 - x))dx > 0;$$

$$\frac{\partial^2 H_2^c}{\partial \bar{s}_2^2} = (h_2 + \beta_2)\phi^{L_2}(\bar{s}_2) + \int_{\bar{s}_2}^{\infty} \phi^{L_2}(x)(G_2''(\bar{s}_1 + \bar{s}_2 - x) + T_1''(\bar{s}_1 + \bar{s}_2 - x))dx > 0.$$

The first inequality reduces to

$$h_1 + h_2 + \alpha p - \iota_1 - \beta_1 > 0.$$

Substituting (8) yields $\iota_1 < h_1 + h_2$ and $\beta_1 < \alpha p$. For the supplier, sufficient conditions are

$$(1 - \alpha)p + \iota_1 + \beta_1 \geq 0;$$

$$h_2 + \beta_2 + (1 - \alpha)p + \beta_1 - ((1 - \alpha)p + \iota_1 + \beta_1)\Phi^{L_1+1}(s_1^o) > 0.$$

Combining the first inequality with (8) yields $\iota_1 \geq 0$ and $\beta_1 \geq -(1 - \alpha)p$. The second inequality, along with (8) and (9), yields $\beta_2 > 0$. □

These are quite reasonable conditions: The first requires that the retailer's inventory subsidy not eliminate retailer holding costs; the second stipulates that the supplier be penalized for its backorders; and the third states that the supplier should not fully reimburse the retailer's backorder costs, and the retailer should not overcompensate the supplier's backorder costs.

To help interpret these results, consider the three extreme contracts where one of the parameters is set to zero:

i) $\iota_1 = 0 \quad \beta_2 = \dfrac{\gamma_2}{1 - \gamma_2} h_2 \quad \beta_1 = -(1 - \alpha)p$

ii) $\iota_1 = h_1 + h_2 \quad \beta_2 = 0 \quad \beta_1 = \alpha p$

iii) $\iota_1 = (1 - \alpha)(h_1 + h_2) \quad \beta_2 = \dfrac{\gamma_2}{1 - \gamma_2} \alpha h_2 \quad \beta_1 = 0.$

(Of these three contracts, the second does not meet the conditions in Theorem 19, because the supplier fully compensates the retailer for all of its costs. The retailer's incentive to choose the optimal policy is weak: s_1^o is a Nash equilibrium strategy, but any $\bar{s}_1$ is too.)

With the first contract the retailer fully reimburses the supplier for the supplier's consumer backorder penalty. However, the supplier still carries inventory because it pays a penalty for its local backorders. With the third contract the supplier subsidizes the retailer's holding costs, but not fully (provided $\alpha > 0$). In addition, the supplier is penalized for its backorders, but less than in the first contract. When the retailer incurs all backorder costs (i.e., $\alpha = 1$), only a supplier backorder penalty is required, $\beta_2 = \gamma_2 h_2/(1 - \gamma_2)$.

Incidentally, (8) and (9) can be written

$$\frac{\alpha p - \beta_1}{(h_1 + h_2)\left(1 - \dfrac{\iota_1}{h_1 + h_2}\right) + (\alpha p - \beta_1)} = \frac{p}{h_1 + h_2 + p}$$

$$\frac{\beta_2}{h_2\left(1 - \dfrac{\iota_1}{h_1 + h_2}\right) + \beta_2} = \gamma_2.$$

Consider the first identity. The quantity $\alpha p - \beta_1$ is the retailer's backorder cost, and its holding cost, including transfer payments, is

$$(h_1 + h_2)\left(1 - \frac{\iota_1}{h_1 + h_2}\right),$$

which is written more simply as $h_1 + h_2 - \iota_1$. So the left-hand side is the retailer's critical ratio. The right-hand side is the critical ratio for the total system costs controlled by the retailer. These ratios must be identical to induce the retailer to minimize total system costs. The second identity specifies a critical ratio for the supplier. β_2 is its (local) backorder cost. The holding cost h_2 is multiplied by the factor $(1 - \iota_1/(h_1 + h_2))$, which is the fraction of actual holding costs paid by the retailer, taking the transfer payment into account. Thus, this rule effectively reduces both stages' holding costs by the same fraction.

7.2. Additional Contracting Issues

Theorem 19 details the contracts that make the optimal solution a Nash equilibrium, but this does not imply a *unique* equilibrium. Nevertheless, even if there were additional Nash equilibria, the one corresponding to the optimal solution Pareto dominates any other. Hence, the players can coordinate on this equilibrium. (There is experimental evidence that players coordinate on a Pareto dominant equilibrium when they are able to converse before playing the game, e.g. Cooper et al. 1989, Cachon and Camerer 1996.)

Although total costs decline when the firms coordinate, one firm's cost may increase. This firm will be unwilling to participate in the contract unless it receives an additional transfer payment. To maintain the strategic balance of the contract, this payment should be independent of all other costs and actions. For example, the firms could transfer a fixed fee each period. Alternatively, one could seek a contract (ι_1, β_2, β_1) such that each firm's cost is no greater than in the original Nash equilibrium.

Finally, this analysis assumes the firms use local base stock levels. In this context local policies have several advantages over echelon policies. Recall that $\bar{s}_1$ has no influence on $\beta_2 B_{2t}$, the supplier's backorder penalty, when firms use local base stock levels. However, with echelon stock base stock levels, s_1 *does* influence $\beta_2 B_{2t}$, holding s_2 constant. This can create a perverse incentive. Suppose β_2 is large. By increasing $s_1 \gg s_2$, the retailer can make B_{2t} arbitrarily large (assuming S, the limit on s_1, is large too). The $\beta_2 B_{2t}$ transfer payment could easily dominate the additional retailer inventory cost. (Furthermore, once $s_1 > s_2$, the retailer can increase s_1 without increasing its inventory.) There is a solution to this problem. The transfer payment could *assume* that the retailer chooses $s_1 \le s_1^o$. Hence, the retailer would receive no additional benefit by raising s_1 above s_1^o. Clearly this increases the complexity of the contract. Local measurements avoid this problem altogether.

8. Numerical Study

The system optimal solution is virtually never a Nash equilibrium, but how large is the difference between their costs? To answer this question, we conducted a numerical study.

One period demand is normally distributed with mean 1 and standard deviation 1/4. (There is only a tiny probability of negative demand.) The remaining parameters are chosen from the 2625 possible combinations of the following:

$$\alpha \in \{0, 0.1, 0.3, 0.5, 0.7, 0.9, 1\} \quad p \in \{1, 5, 25\}$$

$$L_1 \in \{1, 2, 4, 8, 16\} \quad h_1 \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$$

$$L_2 \in \{1, 2, 4, 8, 16\} \quad h_2 = 1 - h_1.$$

Note that $h_1 + h_2 = 1$ in all of the problems; therefore, the retailer holding cost is constant.

For each problem three solutions are evaluated: (1) the system optimal solution; (2) the Nash equilibrium of the EI game; and (3) the Nash equilibrium of the LI game. (These data can be obtained from http://www.duke.edu/˜ gpc/.) (When $\alpha = 1$ there are multiple Nash equilibria in the EI game; we choose the one with the largest s_1.)

Table 1 summarizes the percentage increase in cost of the Nash equilibrium over the system optimal solution. We call this percentage the *competition penalty*. Several results are evident from the table. First, when the players care about backorder costs equally (i.e., $\alpha = 0.5$), the Nash equilibrium is close to the system optimal solution: In the EI and LI games the median competition penalty is 6% and 3%, respectively, and the maximum is 13% and 8%, respectively.

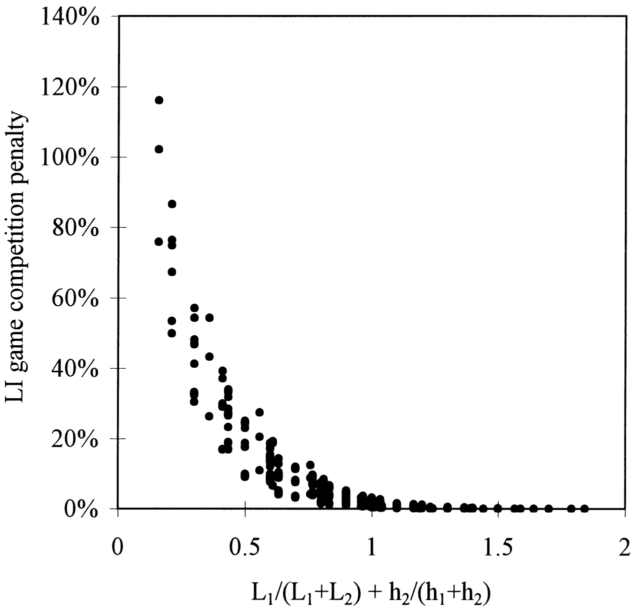
The competition penalty increases as the backorder cost allocation becomes more asymmetric. In the EI game the median percentage is 84% when the retailer incurs no backorder penalty (i.e., $\alpha = 0$) and 483% when the retailer incurs all of the backorder penalty (i.e., $\alpha = 1$). The competitive outcome is poor when $\alpha = 0$ because the retailer chooses $s_1 = \bar{s}_1 = 0$, so all consumer demands are backordered. When $\alpha = 1$, the competition penalty is substantial because the supplier refuses to carry inventory, thereby hampering the retailer's effort to mitigate consumer backorders.

The LI game's response to changes in α is slightly different. The performance of the competitive solution deteriorates rapidly as the retailer incurs lower backorder cost (i.e., α declines). However, as the retailer incurs higher backorder cost (i.e., α increases), the maximum competition penalty increases rapidly, while the minimum and median penalties do not. Consider the extreme case where all backorder costs are allocated to the retailer. In the competitive solution the supplier carries no inventory. Nevertheless, this behavior is not always harmful. Look at Figure 3, which displays the percentage increase in total system costs as a function of

Table 1 The Competition Penalty Under Different Allocations of Backorder Costs

	α	Competition Penalty: Percentage Increase in Cost of the Nash Equilibrium Over the System Optimal Solution				
		Minimum	5th Percentile	Median	95th Percentile	Maximum
Echelon Inventory Game	0	107%	117%	804%	5,930%	10,939%
	0.1	5%	9%	41%	97%	120%
	0.3	2%	4%	12%	21%	27%
	0.5	1%	2%	6%	11%	13%
	0.7	1%	1%	4%	12%	19%
	0.9	1%	1%	8%	40%	66%
	1	2%	15%	483%	9,271%	34,493%
Local Inventory Game	0	107%	117%	804%	5,930%	10,939%
	0.1	5%	8%	37%	96%	119%
	0.3	2%	3%	9%	19%	26%
	0.5	1%	1%	3%	6%	8%
	0.7	0%	0%	1%	4%	9%
	0.9	0%	0%	1%	17%	45%
	1	0%	0%	1%	34%	116%

Figure 3 **Competition Penalty When the Retailer Incurs All Backorder Costs** ($\alpha = 1$)



$$\frac{L_1}{L_1 + L_2} + \frac{h_2}{h_1 + h_2}. \quad (10)$$

What does (10) measure? When $L_1/(L_1 + L_2)$ is large, the supplier's lead time is a small fraction of the total system lead time, so the supplier's decisions have a small impact. When $h_2/(h_1 + h_2)$ is large, there is little benefit to holding inventory at the supplier rather than the retailer. As (10) increases, the supplier's self serving behavior does little damage, therefore the competitive solution's performance is nearly as good as the optimal solution's. Overall, the competition penalty is high when one of the firms has a substantial influence over a major portion of total system costs, but little incentive to help manage that cost.

Table 2 presents data on the percentage change in average supply chain inventory in the two equilibria relative to the optimal solution. Average inventories in the competitive solutions are generally lower than in the optimal solution, except in some cases when the retailer cares little about backorders (α is small). Nevertheless, competition raises supply chain inventory by at most 4%.

9. One Dominant Player

In the LI and EI games the players choose their policies simultaneously. In the Stackelberg version of either game one of the players chooses its base stock level first, announces its choice to the other player, and then the other player chooses its base stock level. As in the EI and LI games, a player is committed to its choice, i.e., the first player cannot change its decision after observing the second player's. We seek sub-game perfect equilibria, i.e., the second player chooses an optimal response to the first player's strategy and the first player (correctly) anticipates this behavior. The Stackelberg version represents a situation where one player is the dominant member of the supply chain (e.g., WalMart, Intel.)

9.1. Supplier Stackelberg Games

The Stackelberg game with the supplier leading is called either the Echelon Inventory Supplier game (EIS) or the Local Inventory Supplier game (LIS), depending on the inventory tracking method. In the EIS (LIS) game the supplier chooses s_2 ($\bar{s}_2$) to minimize its cost, given that it anticipates the retailer will choose $r_1(s_2)$ ($\bar{r}_1(s_2)$). According to the next theorem, there is little difference between the EIS and EI games.

Theorem 20. *When $\alpha < 1$, in the EIS game $\{s_1^a, r_2(s_1^a)\}$ is the unique Stackelberg equilibrium. When $\alpha = 1$, $\{s_1 \in [s_2, S], s_2 \in [0, s_1^a]\}$ are the Stackelberg equilibria.*

Proof. The proof of Theorem 13 shows that if the supplier could choose s_1, it would choose s_1 as large as possible. When $\alpha < 1$, $r_1(s_2) \leq s_1^a$, so the supplier should anticipate $s_1 \leq s_1^a$. Therefore, the supplier chooses $s_2 = r_2(s_1^a)$, and the retailer chooses $s_1 = s_1^a$. When $\alpha = 1$, the supplier wishes to carry no inventory, so it chooses $s_2 \in [0, s_1^a]$, since then the retailer will choose $s_1 \geq s_2$. The supplier cannot choose $s_2 > s_1^a$, because then the retailer chooses $s_1 = s_1^a$, leaving the supplier with some expected inventory. □

In the LIS game the supplier anticipates that the retailer will choose $\bar{r}_1(\bar{s}_2)$. Hence, the supplier's cost is

$$H_2(\bar{s}_2) \overset{def}{=} H_2(\bar{r}_1(\bar{s}_2), \bar{r}_1(\bar{s}_2) + \bar{s}_2)$$
$$= E[\hat{H}_2(\bar{r}_1(\bar{s}_2), \bar{s}_2 - D^{L_2})].$$

Since this is continuous in $\bar{s}_2$, there exists $\bar{s}_2^*$ such that

Table 2 Change in Supply Chain Inventory

	α	Percentage Change in Average Supply Chain Inventory in the Nash Equilibrium Relative to the Optimal Solution: Minimum	5th Percentile	Median	95th Percentile	Maximum
Echelon Inventory Game	0	−18%	−12%	−2%	2%	4%
	0.1	−10%	−7%	−3%	1%	3%
	0.3	−8%	−5%	−2%	0%	0%
	0.5	−13%	−9%	−3%	−1%	0%
	0.7	−22%	−15%	−4%	−1%	0%
	0.9	−37%	−27%	−7%	−1%	−1%
	1	−73%	−56%	−17%	−3%	−2%
Local Inventory Game	0	−18%	−12%	−2%	2%	4%
	0.1	−10%	−7%	−2%	1%	3%
	0.3	−7%	−5%	−2%	0%	1%
	0.5	−10%	−7%	−2%	−1%	0%
	0.7	−18%	−11%	−2%	−1%	0%
	0.9	−34%	−15%	−2%	0%	0%
	1	−38%	−16%	−1%	0%	0%

$H_2(\bar{s}_2^*) = \inf_{\bar{s}_2 \in [0,S]} H_2(\bar{s}_2)$. Hence there exists a Stackelberg equilibrium. Let $\{\bar{s}_1^{ls}, \bar{s}_2^{ls}\}$ be an equilibrium, and let $\{s_1^{ls}, s_2^{ls}\}$ be the equivalent pair of echelon base stock levels, i.e., $s_1^{ls} = \bar{s}_1^{ls}$ and $s_2^{ls} = \bar{s}_1^{ls} + \bar{s}_2^{ls}$.

THEOREM 21. *When $\alpha < 1$, in the LIS game the supplier chooses a base stock level lower than in the LI game, i.e., $\bar{s}_2^{ls} < \bar{s}_2^{l}$ and $s_2^{ls} < s_2^{l}$.*

PROOF. Assuming $\bar{s}_1 = \bar{r}_1(\bar{s}_2)$, differentiate the supplier's cost function with respect to $\bar{s}_2$

$$\frac{dH_2}{d\bar{s}_2} = \frac{\partial H_2}{\partial \bar{s}_1}\bar{r}_1'(\bar{s}_2) + \frac{\partial H_2}{\partial \bar{s}_2} = \left[\Phi^{L_2}(\bar{s}_2)G_2'(\bar{s}_1) + \int_{\bar{s}_2}^{\infty} \phi^{L_2}(x)G_2'(\bar{s}_2 + \bar{s}_1 - x)dx\right] \times \bar{r}_1'(\bar{s}_2) + \frac{\partial H_2}{\partial \bar{s}_2}.$$

From Lemma 6, $\bar{r}_1'(\bar{s}_2) < 0$. When $\alpha < 1$, $G_2'(y) < 0$. So $(\partial H_2/\partial \bar{s}_1)\bar{r}_1'(\bar{s}_2) > 0$. When $\bar{s}_2 \geq \bar{r}_2(\bar{s}_1)$, $\partial H_2/\partial \bar{s}_2 \geq 0$. Hence, for $\bar{s}_2 \geq \bar{r}_2(\bar{s}_1)$, $dH_2/d\bar{s}_2 > 0$. Therefore, $\bar{s}_2^{ls} < \bar{r}_2(\bar{s}_1^{l}) = \bar{s}_2^{l}$. Since $\bar{r}_1(\bar{s}_2) + \bar{s}_2$ is decreasing in $\bar{s}_2$, $\bar{s}_2^{ls} + \bar{r}_1(\bar{s}_2^{ls}) = s_2^{ls} < s_2^{l} = \bar{s}_2^{l} + \bar{r}_1(\bar{s}_2^{l})$. □

9.2. Retailer Stackelberg Games

In the Echelon Inventory Retailer (EIR) and Local Inventory Retailer (LIR) games the retailer anticipates the supplier will choose $r_2(s_1)$ and $\bar{r}_2(\bar{s}_1)$, respectively. Since $r_2(s_1) = \bar{s}_1 + r_2(\bar{s}_1)$ when $s_1 = \bar{s}_1$, the retailer's cost for any base stock level is the same in the two games. Hence, when the retailer is dominant, it is immaterial whether the firms use echelon or local inventory measurements. Existence of an equilibrium is straightforward.

THEOREM 22. *In the EIR and LIR games the retailer chooses a base stock level that is higher than in the EI game (s_1^{e}), but lower than in the LI game (s_1^{l}).*

PROOF. The retailer anticipates that the supplier will choose $r_2(s_1)$. Differentiate the retailer's cost function:

$$\frac{dH_1(s_1, r_2(s_1))}{ds_1} = \frac{\partial H_1(s_1, r_2(s_1))}{\partial s_1} + \frac{\partial H_1(s_1, r_2(s_1))}{\partial s_2} r_2'(s_1) = \Phi^{L_2}(s_2 - s_1)G_1'(s_1) + r_2'(s_1)\int_{s_2 - s_1}^{\infty} \phi^{L_2}(x)G_1'(s_2 - x).$$

Since $r_2'(s_1) > 0$, the above is negative for all $s_1 \le s_1^a = s_1^e$, hence the optimal s_1 is larger than s_1^e. Since $r_2'(s_1) < 1$, and in the LI game equilibrium,

$$\frac{\partial H_1(\bar{s}_1^l, \bar{s}_1^l + \bar{s}_2)}{\partial \bar{s}_1} = \Phi^{L_2}(\bar{s}_2) G_1'(\bar{s}_1^l) + \int_{\bar{s}_2}^{\infty} \phi^{L_2}(x) G_1'(\bar{s}_2 + \bar{s}_1^l - x) = 0,$$

it holds that

$$\frac{dH_1(s_1^l, r_2(s_1^l))}{ds_1} > 0.$$

Hence the retailer chooses a base stock level lower than s_1^l. □

10. Conclusion

When both players care about consumer backorders, the supply chain optimal solution is never a Nash equilibrium, so competitive selection of inventory policies decreases efficiency. Although the players may agree to cooperate and choose supply chain optimal policies, at least one of them has a private incentive to deviate from the agreement. Furthermore, there is a unique Nash equilibrium in either the EI game or the LI game, and these equilibria differ. Hence, while there is little operational distinction between tracking echelon inventory or local inventory (since we assume stationary demand), there is a significant strategic difference. The supplier prefers local inventory, but the retailer's preference depends on the parameters of the game.

In the games we study, competition generally lowers supply chain inventory relative to the optimal solution. In other words, if firms cooperate and choose the optimal solution, they will tend to *increase* inventory. This is a surprising result, since many authors suggest the opposite (e.g., Buzzell and Ortmeyer 1995, Kumar 1996). The rationale is that inventory is a public good: Each firm benefits from more inventory, but each wants the other to invest in it. It is well known that participants tend to underinvest in the provision of public goods (see Kreps 1990). In other settings, cooperation *may* lead to lower inventory. For instance, cooperative firms could share sales information, and this might enable better policies than those available to competitive firms. Nevertheless, inventory remains a public good even here; we suspect there is always a strong tendency for competitive firms to choose lower inventory than in the optimal solution.

Should the players wish to choose the optimal solution cooperatively, we characterize a set of simple linear contracts which eliminate each player's incentive to deviate. These contracts are based on actual inventories and backorders. Implementation of these contracts will not provide dramatic improvements when the players have similar preferences for reducing consumer backorders. We draw this conclusion from a sample of 2625 problems. For each problem we measured the *competition penalty*, the percentage increase in total cost of the Nash equilibrium over the optimal solution. When the players view consumer backorders as equally costly (i.e., $\alpha = 0.5$), the median competition penalty in the EI game is only 6% and in the LI game it is 3%. However, when the players have divergent backorder costs, the competition penalty can be huge. For instance, when the supplier is indifferent to consumer backorders, the median competition penalty in the EI game is 483%. These results highlight an important lesson for managers: While the lack of cooperation/coordination implies the system will not perform at its best efficiency, the magnitude of the efficiency loss is context specific.[1]

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